1. Let $f(x) = \sin(x)$

   (a) Recall that the Maclaurin series for $f(x)$ is $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$. For what values of x does the Maclaurin series converge?

   (b) Suppose we want to use a partial sum of the Maclaurin series to estimate $\sin(\pi/3)$. What is $T_5(x)$?

   (c) What is $f^{(6)}(x)$?

   (d) Recall the Maclaurin series is a Taylor series centered at $a = 0$. If $|x - 0| \le \pi/3$, what is the maximum value of $|f^{(6)}(x)|$?

   (e) In Taylor's inequality, the maximum found in part (d) is called M. So we have,

   $$|R_5(\pi/3)| \le \frac{M}{(5+1)!} |\pi/3 - 0|^{5+1}$$

   Plug in your value for M and simplify to find an upper bound for $|R_5(\pi/3)| = |T_5(\pi/3) - \sin(\pi/3)|$.

2. Let $f(x) = e^x$. Suppose we want to calculate $e^{-0.2}$ correctly to within five decimal places using the Maclaurin series for e^x (Note: Within 5 decimal places means $|\text{error}| \le 0.000005$.)

   (a) What is $f^{(n)}(x)$ for a given integer n?

   (b) Find the maximum value of $f^{(n+1)}(x)$ if $|x - 0| \le 0.2$. This is M.

   (c) Set up Taylor's inequality for $x = -0.2$ and a general n.

   (d) We want $|R_n(-0.2)| \le 0.000005$. Find the smallest value of n that guarantees this using technology or guess-and-check.

   (e) Find $T_n(-0.2)$ for the value of n you found in (d). This is our estimate for $e^{-0.2}$, correct to 5 decimal places.

3. Suppose we want to estimate $\int_0^1 x \cos(x^3)\, dx$ to within three decimal places ($|\text{error}| \le 0.0005$).

   (a) Use the Maclaurin series $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ to find the Maclaurin series for $f(x) = x\cos(x^3)$.

   (b) Find the general antiderivative $F(x) = \int x \cos(x^3)\, dx$ by integrating the Maclaurin series for $f(x)$.

   (c) Remember $\int_0^x f(t)\, dt = F(x) - F(0)$. Show $F(0) = 0$. Then $\int_0^1 f(t)\, dt = F(1)$.

   (d) We have a Maclaurin series for $F(x)$. Using this series, what is $F^{(n+1)}(x)$ for a given n?

   (e) What is the maximum value of $|F^{(n+1)}(x)|$ for $|x - 0| \le 1$?

   (f) Use Taylor's Inequality with the value of M you just found to find the number of terms necessary for $|R_n(1)| \le 0.0005$. (Guess-and-check or graph.)

   (g) Find the partial sum for $F(1)$ for the value of n you just found. This is an estimate for $\int_0^1 x \cos(x^3)\, dx$ to within three decimal places.